

The Cell Cycle Switch Computes Approximate Majority

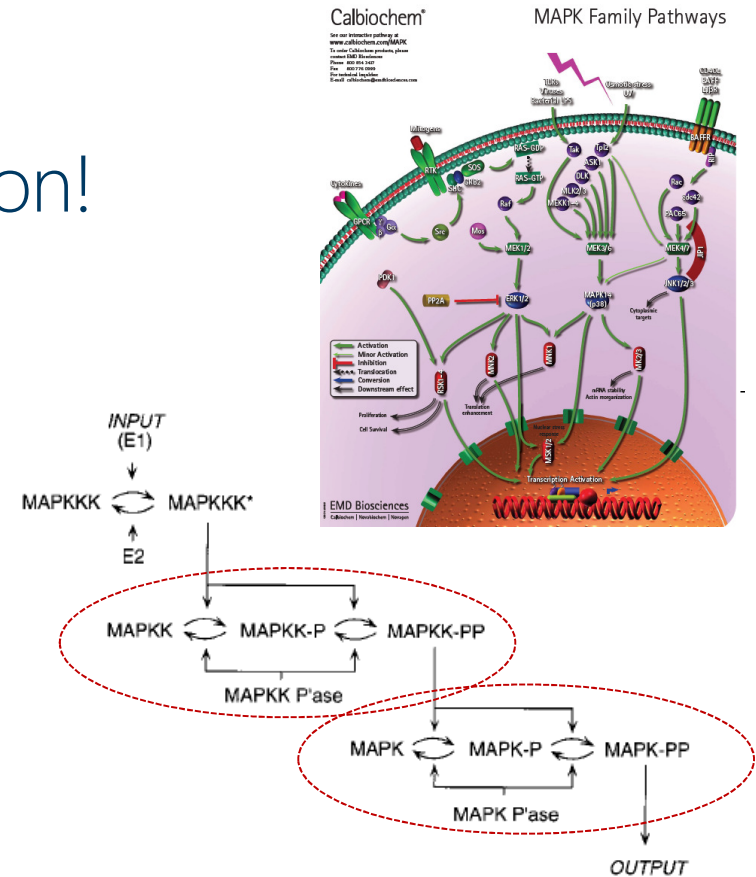
Luca Cardelli, Microsoft Research & Oxford University

with Attila Csikász-Nagy, King's College London

Caltech, 2015-06-02

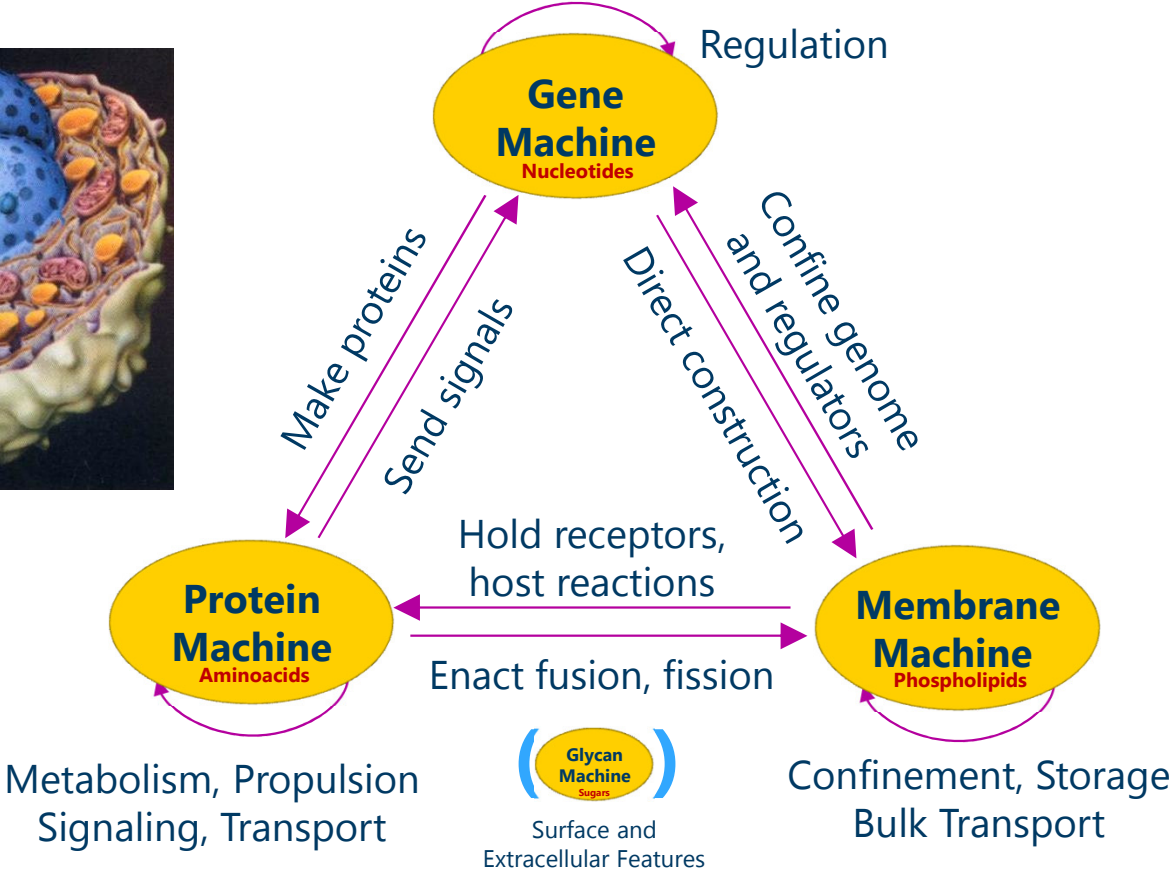
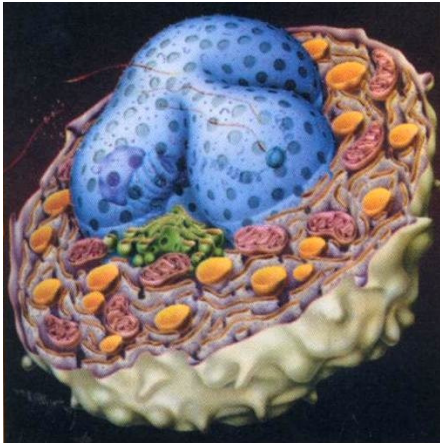
Cellular Computation

- No survival without computation!
 - Finding food
 - Avoiding predators
- How do cells compute?
 - *Clearly* doing “information processing”
 - What are their computational principles?

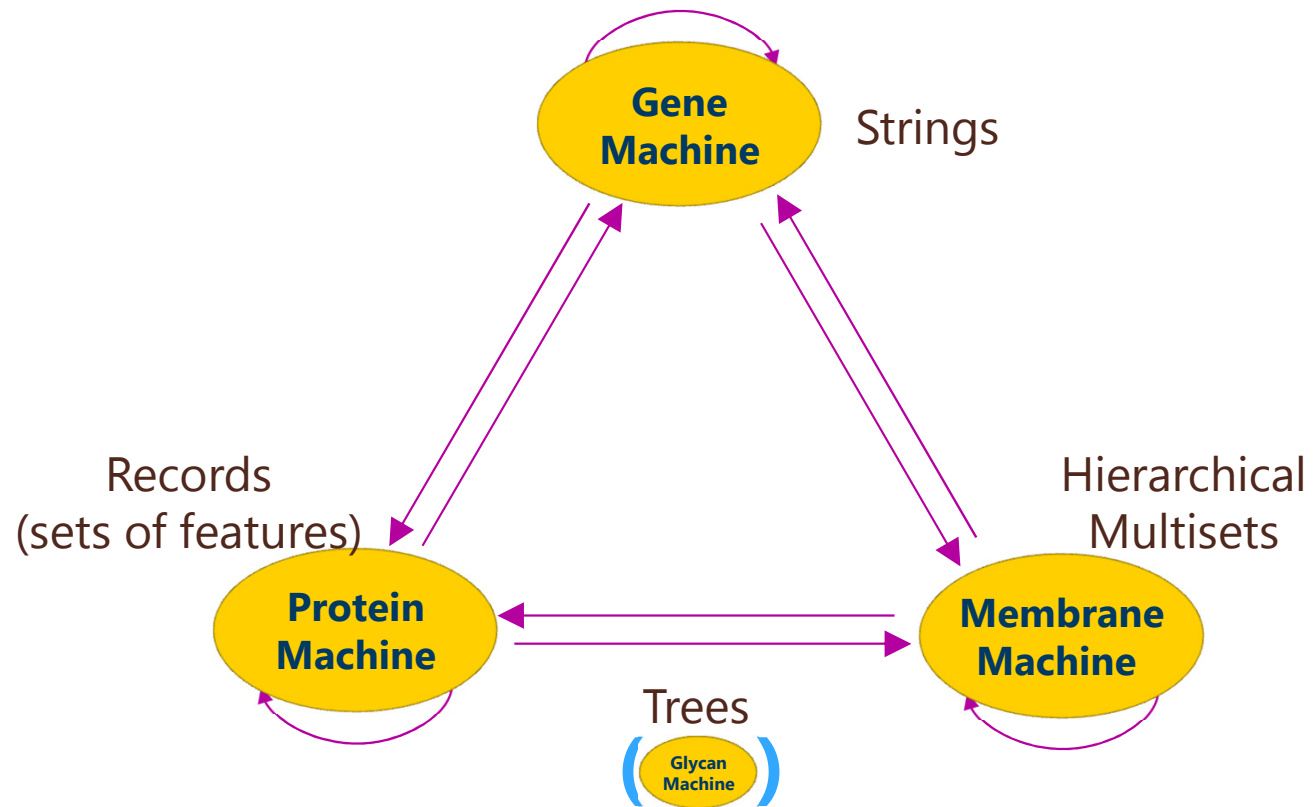


[Ultrasensitivity in the mitogen-activated protein cascade](#), Chi-Ying F. Huang and James E. Ferrell, Jr., 1996, *Proc. Natl. Acad. Sci. USA*, 93, 10078-10083.

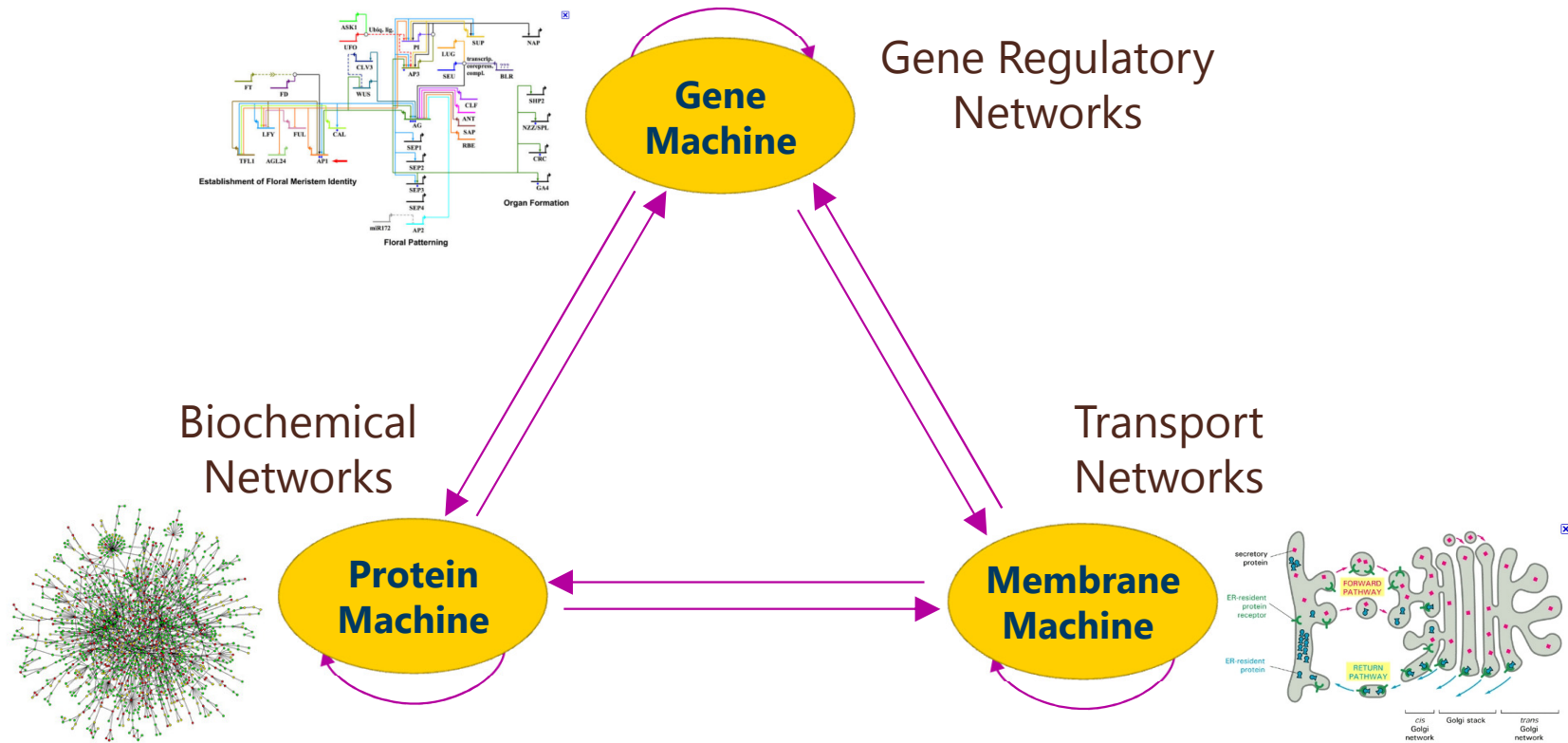
Abstract Machines of Biochemistry



Bioinformatics View (Data Structures)

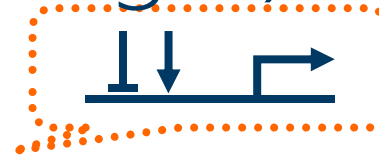
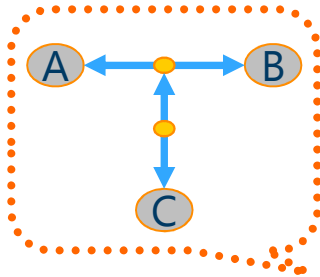


Systems Biology View (Networks)



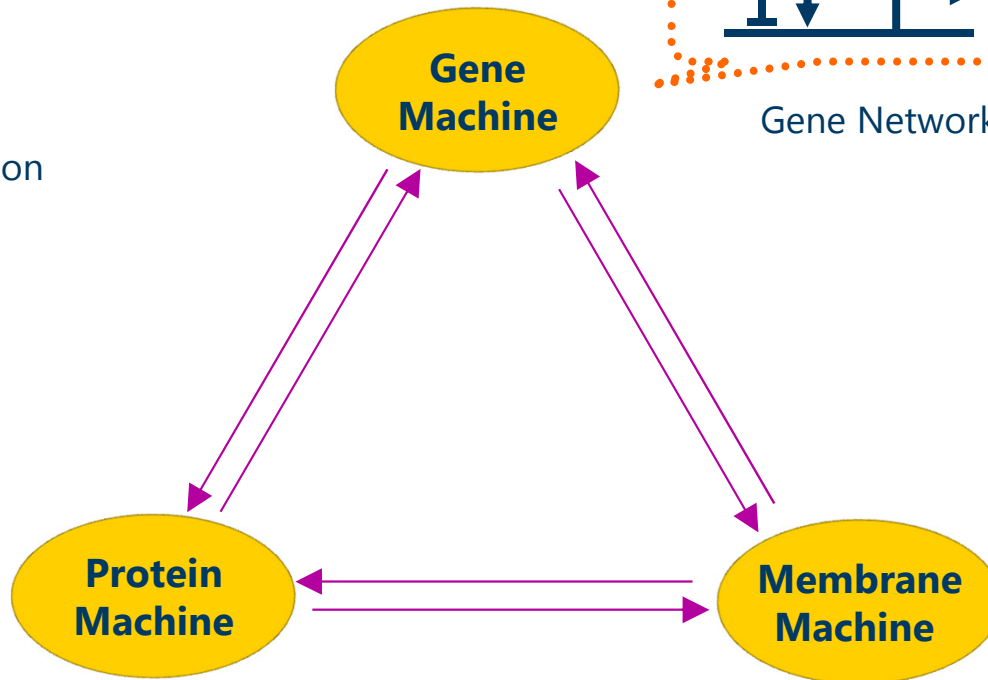
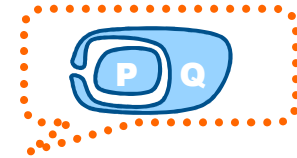
Algorithmic View (Languages)

Molecular Interaction Maps



Gene Networks

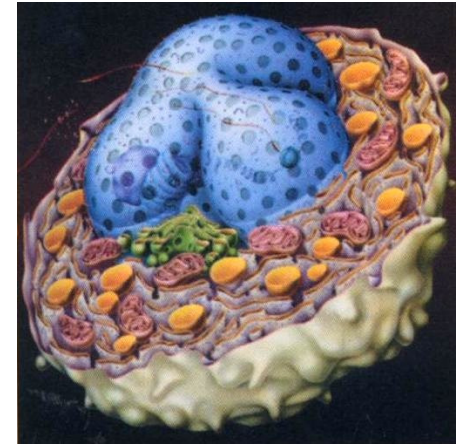
Transport Networks



**These 3 machines
are Turing powerful!**

More concretely

- Give substance to the claim that “cells compute”
 - Yes, but *what* do they compute?
- Catch nature red-handed in the act of running a computational task
 - Something that a computer scientist would recognize as an *algorithm*



Chemical Algorithms

Can *Chemistry* Compute?

- If we believe that biology can do computation...
 - It must be somehow based on chemistry
- So, can chemistry compute, and how?
 - That is in itself a very interesting question with non-trivial answers

Chemical Programming Examples

specification

$Y := \min(X1, X2)$

$Y := \max(X1, X2)$

program

$X1 + X2 \rightarrow Y$

$X1 \rightarrow L1 + Y$

$X2 \rightarrow L2 + Y$

$L1 + L2 \rightarrow K$

$Y + K \rightarrow 0$

$\max(X1, X2) =$
 $(X1 + X2) - \min(X1, X2)$

(but is not computed
"sequentially": it is a form
of concurrent computation)

A Consensus Problem

- Population Consensus

- Given two populations of x and y “agents”
- we want them to “reach consensus”
- by converting *all* agents to x or to y depending on which population was in majority initially

- Population Protocols Model

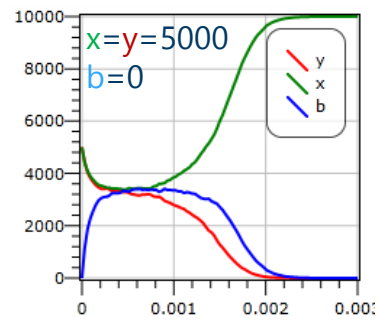
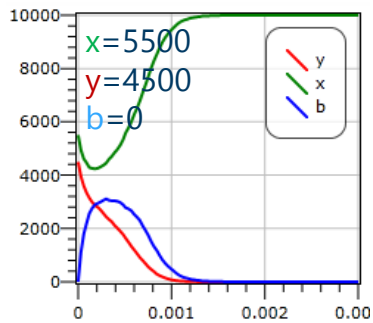
- Finite-state identity-free agents (**molecules**) interact in randomly chosen pairs (\Rightarrow stochastic symmetry breaking)
- Each interaction (**collision**) can result in state changes
- Complete connectivity, no centralized control (**well-mixed solution**)

specification

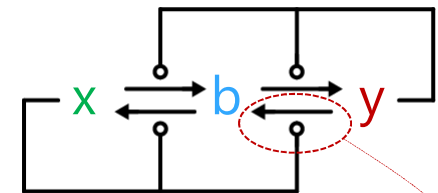
$$\begin{aligned} X, Y &:= X+Y, 0 && \text{if } X_0 \geq Y_0 \\ X, Y &:= 0, X+Y && \text{if } Y_0 \geq X_0 \end{aligned}$$

A Consensus Algorithm

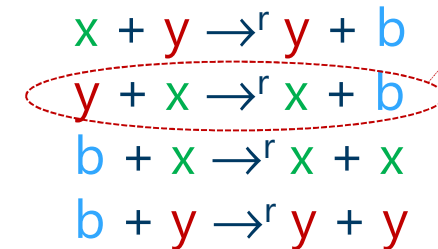
- Approximate Majority (AM) Algorithm
 - Uses a third "undecided" population b
 - Disagreements cause agents to become undecided
 - Undecided agents agree with any non-undecided agent



catalysis \circ



chemical
reaction
network

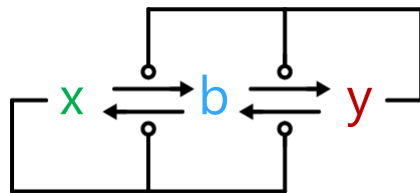


Dana Angluin · James Aspnes · David Eisenstat

A Simple Population Protocol for Fast Robust
Approximate Majority

A Biological Implementation

Approximate Majority (AM)



- 1) **Bistable**
Even when initially $x=y$ (stochastically)
- 2) **Fast (asymptotically optimal)**
 $O(\log n)$ convergence time
- 3) **Robust to perturbation**
above a threshold, initial majority wins *whp*

Dana Angluin · James Aspnes · David Eisenstat

A Simple Population Protocol for Fast Robust Approximate Majority

2007

Epigenetic Switch

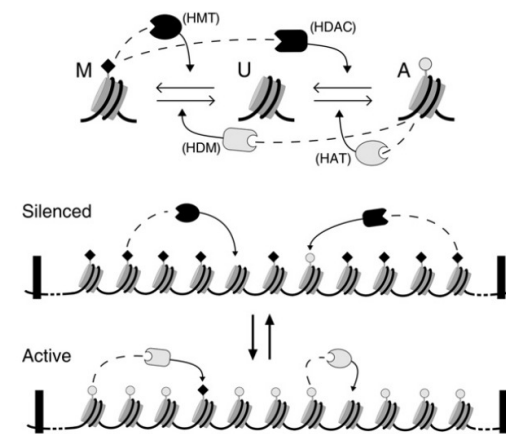


Figure 1. Basic Ingredients of the Model

Theory

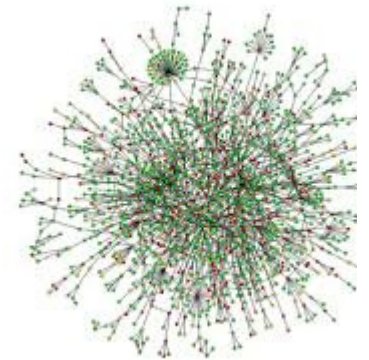
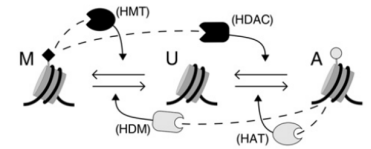
Theoretical Analysis of Epigenetic Cell Memory by Nucleosome Modification

Jan B. Dückel,^{1,2} Maja A. Michielescu,¹ Kim Sjögreen,^{1,2} and Genevieve Thorpe¹
¹Center for Molecular Life Mechanics Institute, Biogenetisk Center, Copenhagen N, Denmark
²Department of Molecular and Biomedical Science, Biochemistry, University of Adelaide SA 5005, Australia
³Department of Molecular Biology, University of Copenhagen, Biocenter, Ole Warburgs Vej 5, DK-2200 Copenhagen N, Denmark
 Correspondence: jueduckel@bmc.au.dk
 DOI: 10.1101/041207 (2007)

2007

Here We Got Lucky

- We can claim that the epigenetic switch is a *direct* biological implementation of an algorithm
 - Although we may have to qualify that with some notion of approximation of the (enzymatic) kinetics
- In most cases the biological implementation seems more *indirect* or *obfuscated*
 - "Nature is subtle but not malicious - Einstein" Ha! think again!
 - Other implementations of Approximate Majority seem more convoluted and... approximate

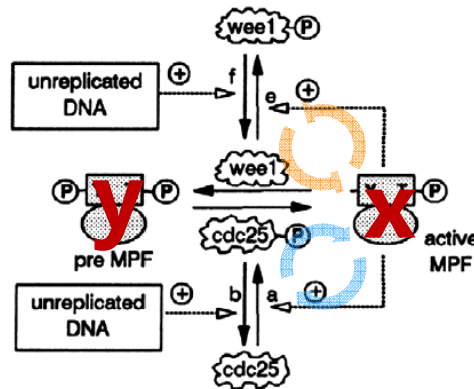


The Cell Cycle Switch

Universal control mechanism regulating onset of M-phase

Paul Nurse

- This basic network is **universal in Eukaryotes** [P. Nurse]
 - The *switching function* and the *basic network* is *the same* from yeast to us.
 - In particular detail, in frog eggs:



Double positive feedback on x
 Double negative feedback on x
 No feedback on y. Why ???

Journal of Cell Science 106, 1153-1168 (1993)
 Printed in Great Britain © The Company of Biologists Limited 1993

Numerical analysis of a comprehensive model of M-phase control in *Xenopus* oocyte extracts and intact embryos

Bela Novak* and John J. Tyson†

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*Permanent address: Department of Agricultural Chemical Technology, Technical University of Budapest, 1521 Budapest Gellert Ter 4, Hungary

†Author for correspondence

- The function is very well-studied. But why this network structure?
- That is, *why this peculiar algorithm?*

How to model "Influence"

"True" molecular interactions.

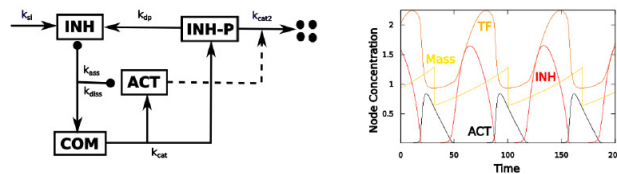


Figure 3: a) Schematic diagram of a simplified SIMM model [17]. The activa-

"Equivalent" influence interactions.

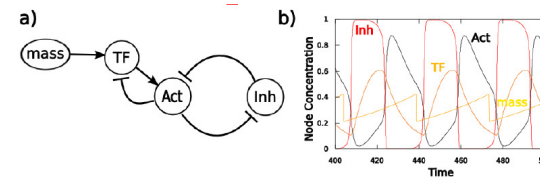


Figure 4: a) Schematic diagram of a primitive cell cycle in the reinitz framework.

Chemical Reaction Network



Influence Network

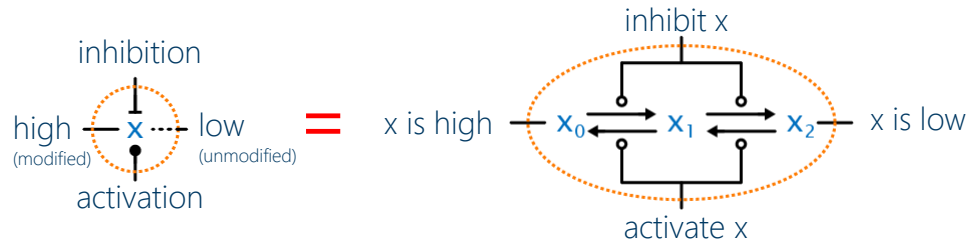
Evolving a Primitive Eukaryotic Cell Cycle Model

Malte Lücken, Jotun Hein, Bela Novak

Instead of modeling basic interactions, such as binding, synthesis, and degradation of molecular components, this framework models interactions simply as activation or inhibition. This approach also reduces the number of nodes necessary in the network, as e.g. the inhibitor binding tightly to the activator to form a complex, which produces phosphorylated inhibitor to be degraded under catalysis by the activator, is now simply a double negative feedback loop shown in Figure 1. This type of interaction is the basis of both aforementioned molecular model, therefore they can both be summarized in a single Reinitz model.

The Triplet Model of Influence

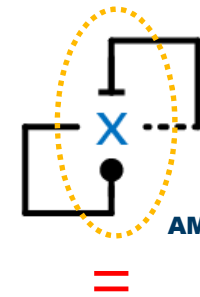
activation ●
 inhibition T
 catalysis ○



triplet motif

We model them by
 4 mass action reactions over
 3 species x_0, x_1, x_2

For example:



Usually modeled by
 sigmoid (e.g. Hill or
 Reinitz) functions

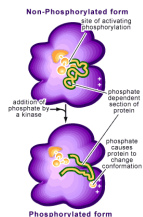


Functional Motifs in
 Biochemical Reaction
 Networks
 John J. Tyson¹ and Bela Novak²

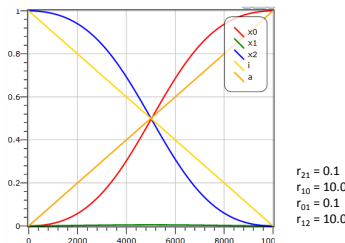
$$\frac{dx_i}{dt} = \gamma_i \frac{[A_i(1-x_i) - B_i x_i]}{A_i + B_i}, \quad i = 1, \dots, N.$$

$$A_i = \exp\left\{\sigma_i \left(\alpha_{i0} + \sum_{j=1}^N \alpha_{ij} X_j\right)\right\}, \quad B_i = \exp\left\{\sigma_i \left(\beta_{i0} + \sum_{j=1}^N \beta_{ij} X_j\right)\right\}.$$

biological mechanism:
 (e.g.): multisite
 phosphorylation



They actually implement a
 Hill function of coefficient 2:



Approximate Majority

How to Build a Good Switch

- We need first a **bistable** system: one that has two *distinct* and *stable* states. I.e., given any initial state the system must settle into one of two states
- The settling must be **fast** (not get stuck in the middle for too long) and **robust** (must not spontaneously switch back)
- Finally, we need to be able to **flip** the switch by external inputs

A Bad Algorithm

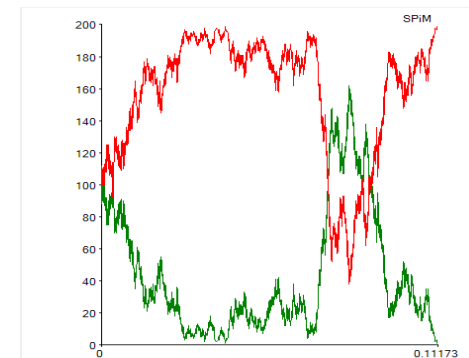
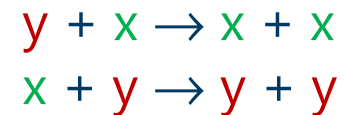
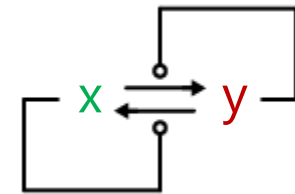
- Direct Competition

- x catalyzes the transformation of y into x
- y catalyzes the transformation of x into y
- when all-x or all-y, it stops

- This system has two end states, but

- Convergence to an end state is slow (a random walk)
- Any perturbation of an end state can start a random walk to the other end state (hence not really *bistable*)

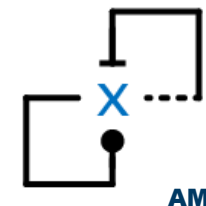
catalysis 



A Good Algorithm

- Approximate Majority (AM)
 - Third, undecided, state b
 - Disagreements cause agents to become undecided
 - Undecided agents believe any non-undecided agent
- With high probability, for n agents
 - The total number of interactions before converging is $O(n \log n)$
⇒ fast (optimal)
 - The final outcome is correct if the initial disparity is $\omega(\sqrt{n} \log n)$
⇒ solution states are robust to perturbations
- Logarithmic time bound in parallel time
 - *Parallel time* is the number of steps divided by the number of agents
 - In parallel time the algorithm converges with high probability in $O(\log n)$

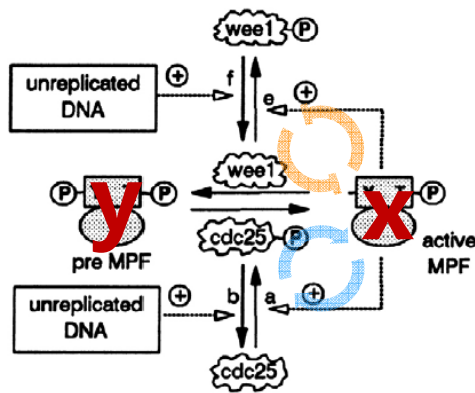
activation ●
inhibition ⊣



Dana Angluin · James Aspnes · David Eisenstat

A Simple Population Protocol for Fast Robust
Approximate Majority

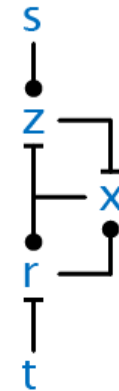
An "Ugly" Algorithm: Cell Cycle Switch



Nobel-prize winning network

activation ●
inhibition ⊥

Obfuscation of a distributed algorithm?

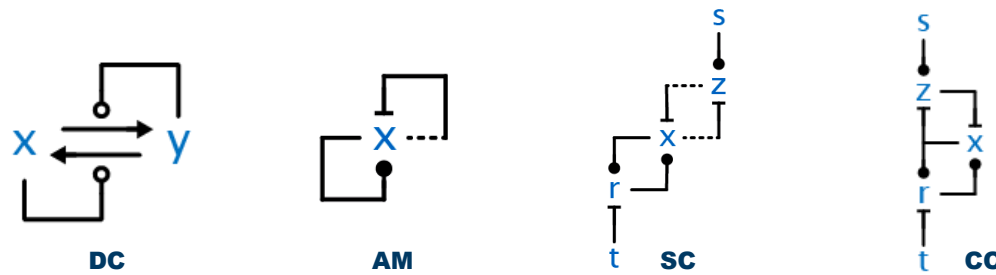


- Is it a good algorithm? Is it bad?
- Is it optimal or suboptimal?

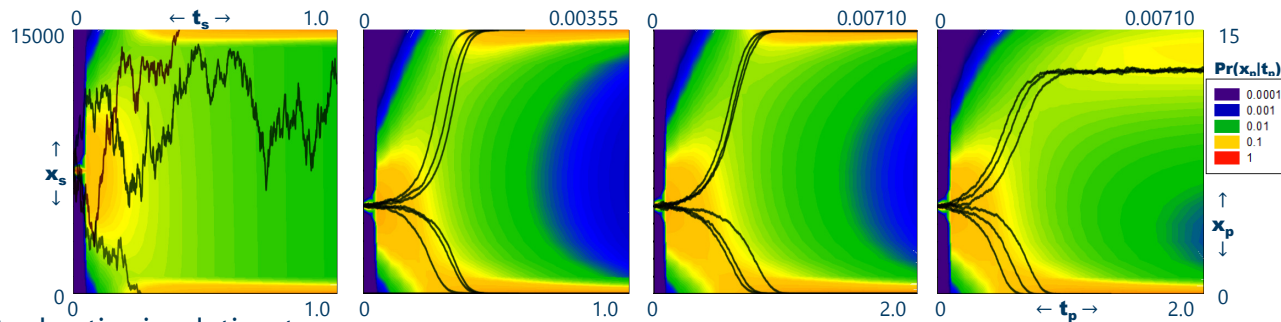
Convergence Analysis

- Switches as **computational systems**

CC converges in $O(\log n)$ time (like AM)
(but 2x slower than AM, and does not fully switch)



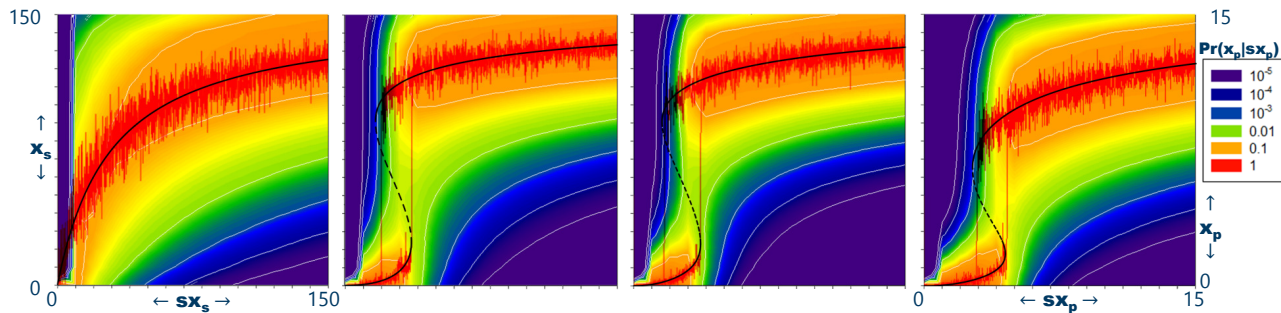
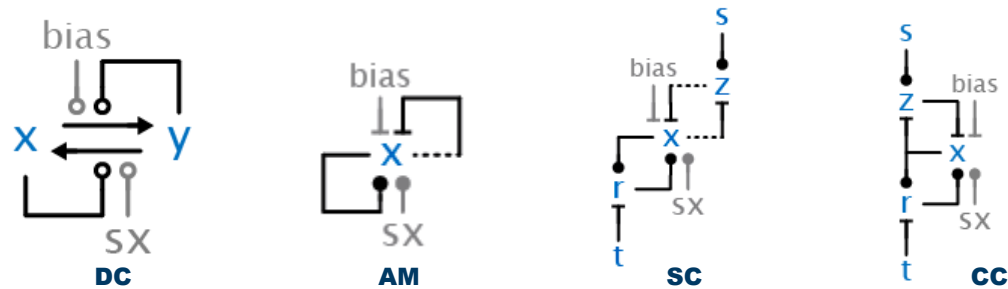
Start symmetrical
($x_0 = x_1 = x_2$ etc.)



Black lines: several stochastic simulation traces
Color: full probability distribution of small-size system

Steady State Analysis

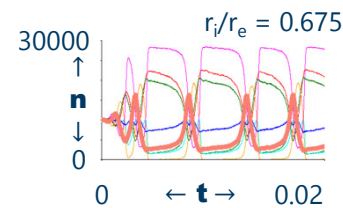
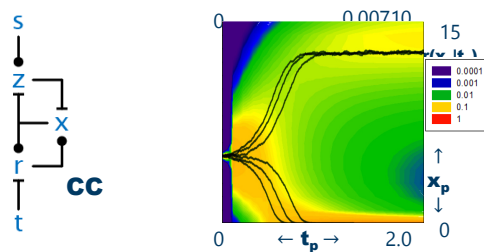
- Switches as dynamical systems



Black lines: deterministic ODE bifurcation diagrams
 Red lines: noisy stochastic simulations
 Color: full probability distribution of small-size system

Why is CC worse than AM?

- The classical CC has an algorithmic “bug”
 - It works ok but never as well as AM
 - Because s continuously inhibits x through z , so that x cannot fully express

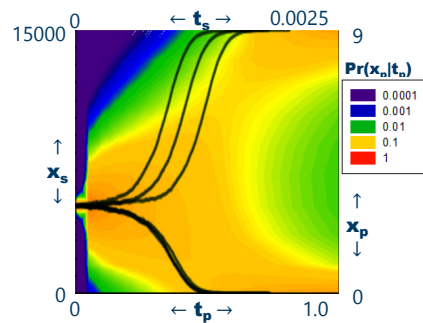
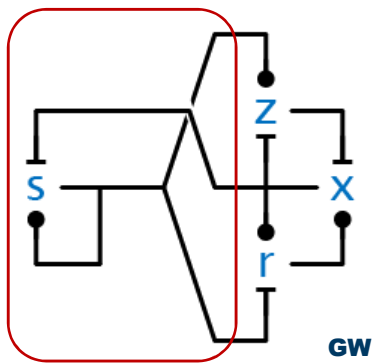


The corresponding cell cycle oscillator is also depressed

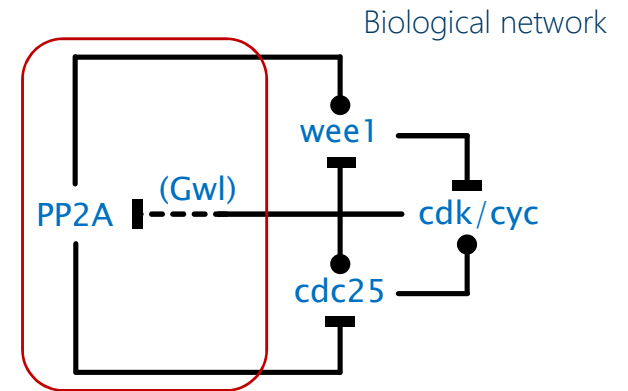
- So let's fix the bug!
 - Easy: let x inhibit s and t “in retaliation”
 - Q: Why didn't nature fix it?

Nature fixed it!

- There is another known feedback loop
 - By which x suppresses s "in retaliation" via the so-called **Greatwall** loop
 - Also, s and t happen to be the same molecule ($=s$)



Full activation!



- s and x now are antagonists: they are **the two halves of the switch**, mutually inhibiting each other (through intermediaries).

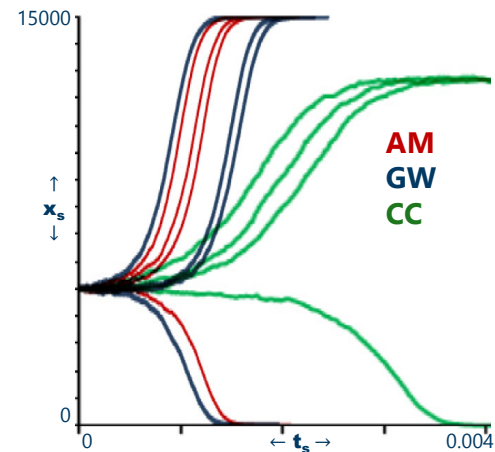
More surprisingly

- The fix makes it faster too!
 - The extra feedback also speeds up the decision time of the switch, making it about as good as the 'optimal' AM switch:

Conclusion:

Nature is trying as hard as it can to implement an AM-class algorithm!

The "classical" cell cycle switch is only half of the picture: the extra feedback completes it *algorithmically*.



Publications

- Our paper appeared:
 - Suggesting GW is a better switch than CC. *September 2012*
- Another paper that same week:
 - Showing experimentally that the Greatwall loop is a **necessary** component of the switch, i.e. the not-as-good-as-AM network has been 'refuted'

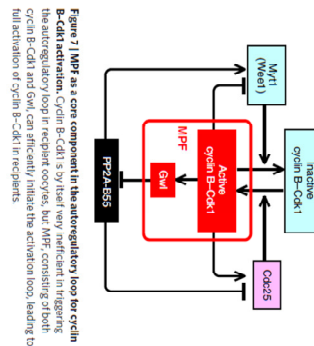
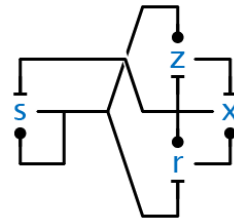


Figure 7 | MPF as a core component in the autoregulatory loop for cyclin B-Cdk1 activation. Cyclin B-Cdk1 is by itself very inefficient in triggering the autoregulatory loop in recipient oocytes, but MPF, consisting of both cyclin B-Cdk1 and Gwl, can efficiently initiate the activation loop, leading to full activation of cyclin B-Cdk1 in recipients.



The Cell Cycle Switch Computes Approximate Majority

SUBJECT AREAS:
COMPUTATIONAL
BIOLOGY

Luca Cardelli¹ & Attila Csikász-Nagy^{2,3}



ARTICLE

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DOI:10.1038/ncomms2062

Greatwall kinase and cyclin B-Cdk1 are both critical constituents of M-phase-promoting factor

Masatoshi Hara^{1,1}, Yusuke Abe^{1,1}, Toshiaki Tanaka², Takayoshi Yamamoto^{1,1}, Eiichi Okumura¹ & Takeo Kishimoto¹

But again, is CC (or GW) the “same” as AM?

- Our evidence for computational content of biochemical networks is so far
 - Quantitative, covering both kinetic and steady state behavior of *what* networks do
 - But empirical (based on simulations/numerical solutions)
 - And it does not yet explain *how* the CC/GW network relates to the AM network, that is, how each *piece* of CC/GW corresponds to each *piece* of AM
- Analytical evidence is harder to obtain
 - The proofs of the computational properties (optimality etc.) for the AM algorithm are hard and do not generalize easily to more complex networks
 - Quantitative theories of behavioral equivalence and behavioral approximation, e.g. in process algebra, are still lacking (although rich qualitative theories exist)

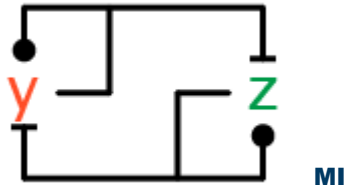
Network Morphisms

When does a (complex) network
implement a (simpler) algorithm?

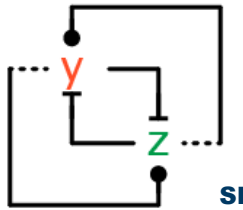
Antagonistic Networks

activation ●
inhibition ⊣

1 vs. 1
Mutual Inhibition &
Self Activation

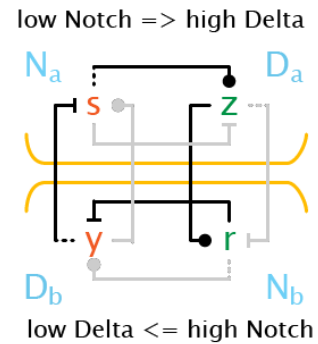


1 vs. 1
Mutual Inhibition &
Mutual Anti-activation

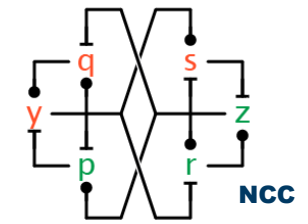


2 vs. 2
low Notch => high Delta
low Delta =<= high Notch

low Delta => low Notch
high Delta => high Notch



3 vs. 3



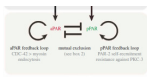
Cell cycle transitions

Molecular mechanisms creating bistable switches at cell cycle transitions
Ansel Vergara, P. K. Maini, John J. Tyson and Bela Novak
Open Biol 2013, 9: 130174, published 15 March 2013



Polarity establishment

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY
The PAR network redundancy and robustness in a symmetry-breaking system
Ferdie Merys^{1,2} and Caroline Saffell¹
¹Thames Valley University, 'Theoretical Biology and Biophysics', School of Biological Sciences, Norfolk University of Applied, Norwich, UK, ²Open 1706, Faculty of Applied Sciences, University of East Anglia, Norwich, UK, ³Open 1706, Faculty of Applied Sciences, University of East Anglia, Norwich, UK

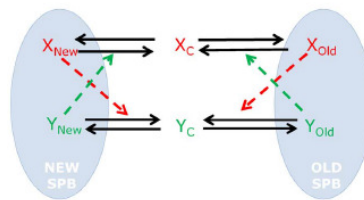


Gene networks

Construction of a genetic toggle switch in *Escherichia coli*
Timothy S. Gardner^{1,2}, Charles R. Cantor¹ & James J. Collins^{1,2}



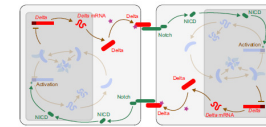
Septation Initiation



Dynamics of SIN Asymmetry Establishment

Anshu Rajan¹, Arno Fackeldey², Jun-Sung Cha³, Daniel McCollum¹, Massimo Santilli^{1,4}, Ralf G. Curjel^{1,5}, Ashwin L. Grodz¹, Arne Kohler^{1,6}
¹Open 1706, ²Open 1706, ³Open 1706, ⁴Open 1706, ⁵Open 1706, ⁶Open 1706

Delta-Notch



Patterning embryos with oscillations: structure, function and dynamics of the vertebrate segmentation clock
Andrew C. Gaten¹, Luis G. Morelli² and Sall Aze^{1,4}

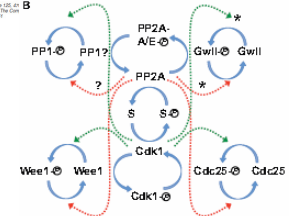
Lateral Inhibition through Delta-Notch Signaling: A Piecewise Affine Hybrid Model

Ranjay Ghosh and Chao J. Tomlin
SIAM J. Appl. Math. 2010, 70: 1000-1015, published 2010

The "new" cell cycle switch

Phosphorylation network dynamics in the control of cell cycle transitions
Daniel Fisher¹, Liliana Krasinska^{1,2}, Damien Coudreuse^{1,3} and Bela Novak^{1,4}

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²Unité de Génétique Moléculaire de Strasbourg, CNRS, ILL, USC, Université de Strasbourg, France
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New Cell Cycle Switch Network

- A recent paper presents a more complete view of the cell cycle switch
- N.B. “phosphorylation network dynamics” here is the same as our x_0 - x_1 - x_2 motif

Phosphorylation network dynamics in the control of cell cycle transitions

Daniel Fisher^{1*}, Liliana Krasinska^{1,2}, Damien Coudreuse^{2,3} and Béla Novák^{3,2}

¹Institut de Génétique Moléculaire de Montpellier, IGMM, CNRS UMR 5535, Université Montpellier I and II, 34293 Montpellier, France

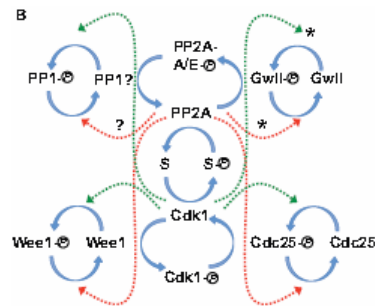
²Institute of Genetics and Development of Rennes, CNRS UMR 6290, 35043 Rennes, France

³Oxford Centre for Integrative Systems Biology, Department of Biochemistry, University of Oxford, South Parks Road, Oxford OX1 3OU, UK

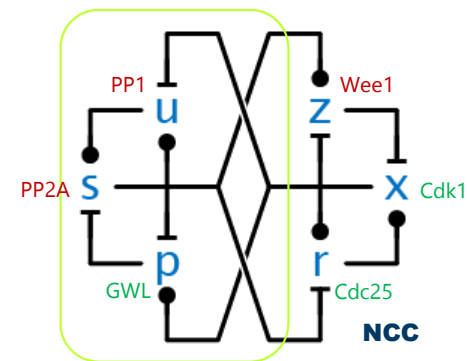
*Author for correspondence (daniel.fisher@igmm.cnrs.fr)

[†]These authors contributed equally to this work.

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doi: 10.1242/jcs.10651

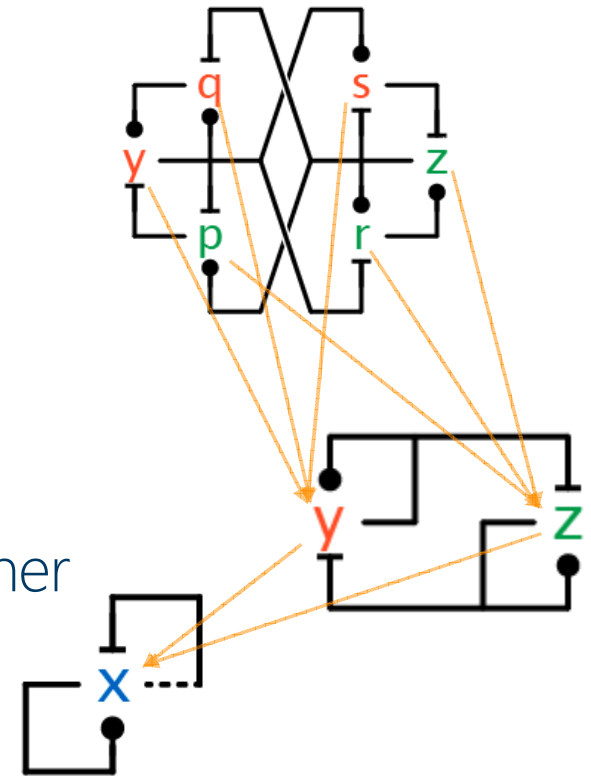


Mutual inhibition between *three* species each



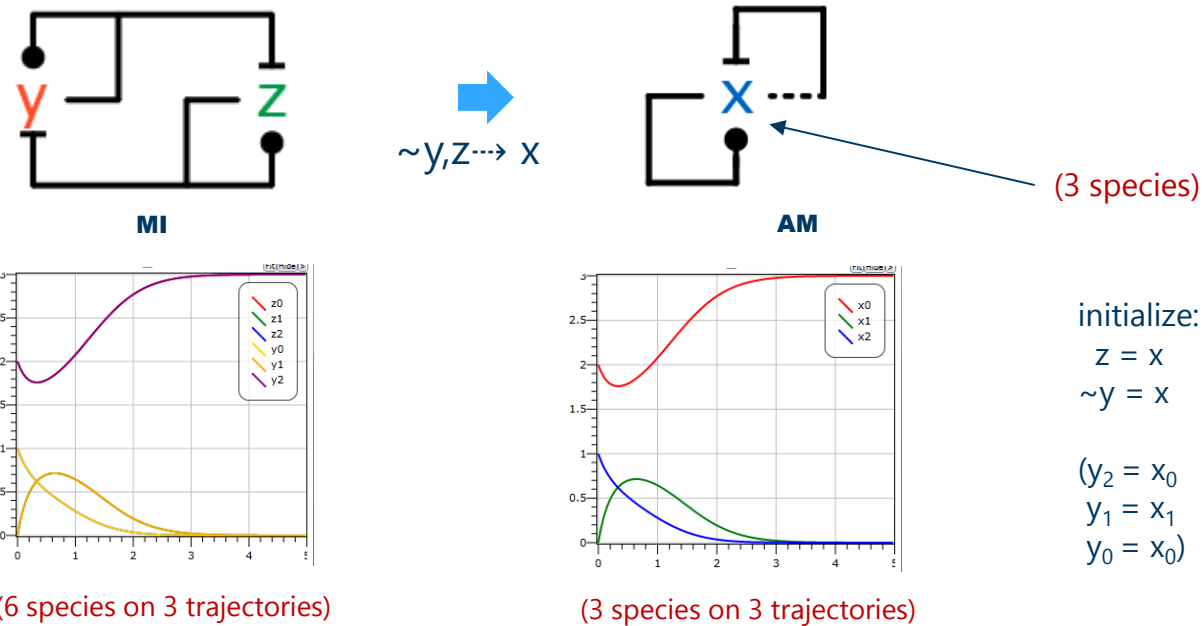
Comparing networks

- How can we compare different networks?
 - Different number of species
 - Different number of reactions
 - Apparently unrelated connectivity
- So that we can compare their function?
 - Does antagonism (in network structure) guarantee bistability (in function)?
- We do it by *mapping* networks onto one another so that they *emulate* each other



Network Emulation: MI emulates AM

- For **any rates and initial conditions** of AM, we can find *some* rates and initial conditions of MI such that the (6) trajectories of MI retrace those (3) of AM:



- How do we find these matching parameters? By a **network morphism!**

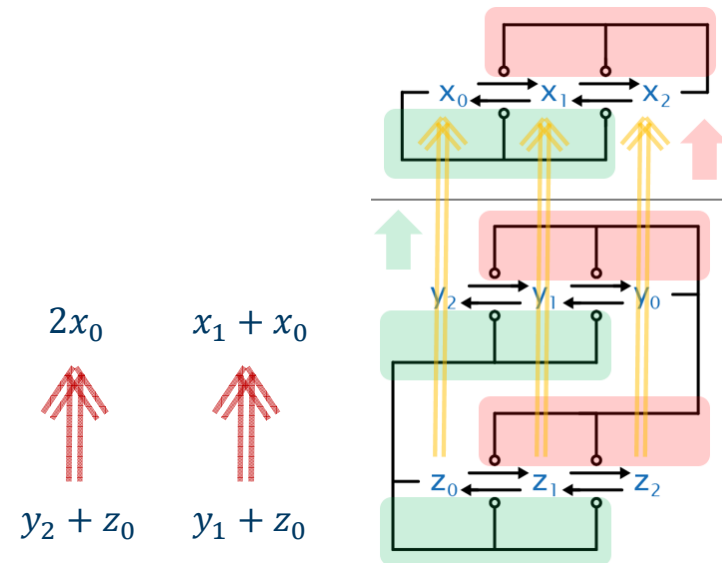
CRN Morphisms

A *CRN morphism* from (S, R) to (\hat{S}, \hat{R})
 written $m \in (S, R) \rightarrow (\hat{S}, \hat{R})$

is a pair of maps $m = (m_S, m_R)$
 a species map $m_S \in S \rightarrow \hat{S}$
 a reaction map $m_R \in R \rightarrow \hat{R}$

extended to a complex map $m_S \in \mathbb{N}^S \rightarrow \mathbb{N}^{\hat{S}}$
 linearly: $m_S(\rho)_{\hat{s}} = \sum_{s \in m_S^{-1}(\hat{s})} \rho_s$

Mappings (symmetries)
 between two networks

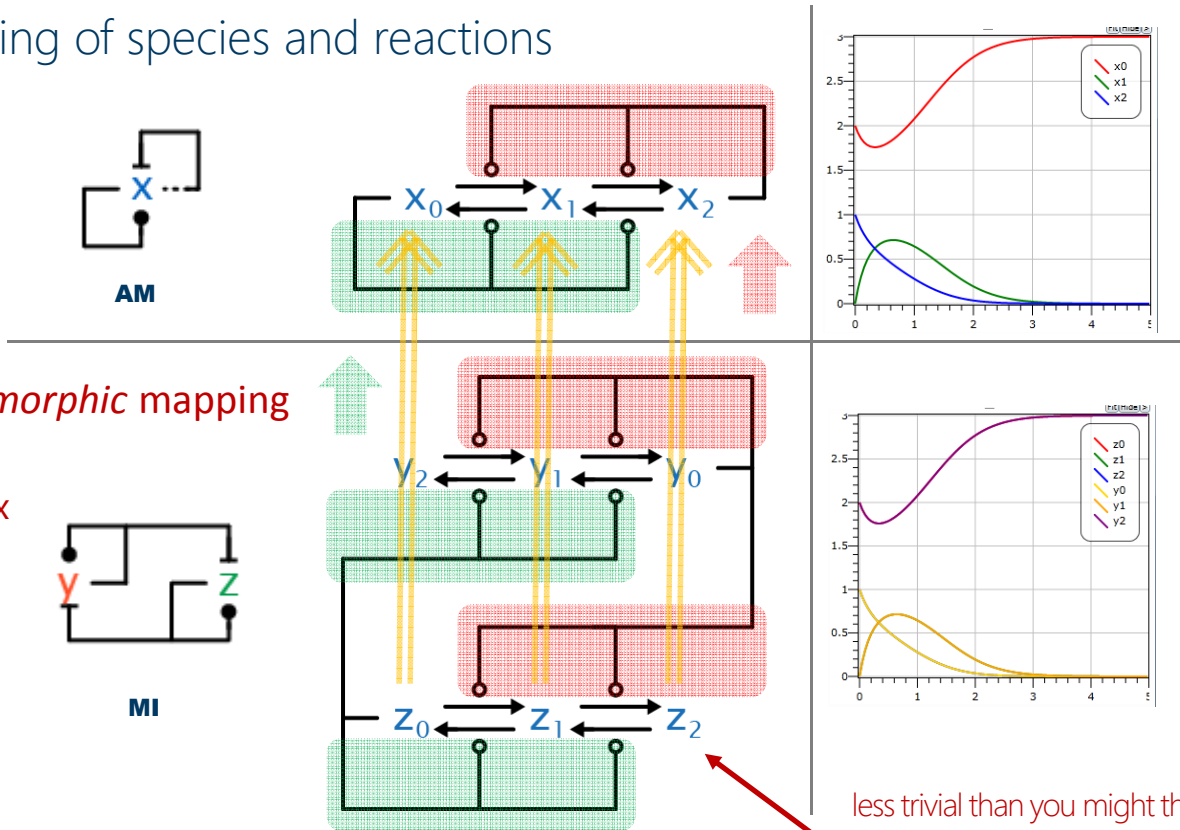


How to check emulations

- How do we check a potential emulation morphism **for all possible initial conditions** of the target?
 - Statically: check conditions on the joint stoichiometric matrices of the two networks under the mapping
- How do we check a potential emulation morphism **for all possible rates** of the target?
 - Can't; but if one emulation is found, then the rates of the target network can be changed *arbitrarily* and a related emulation will again exist

Network Emulation: MI emulates AM

A mapping of species and reactions



any initial conditions

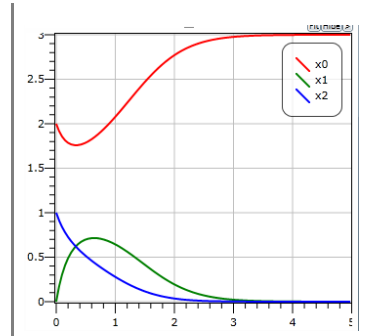
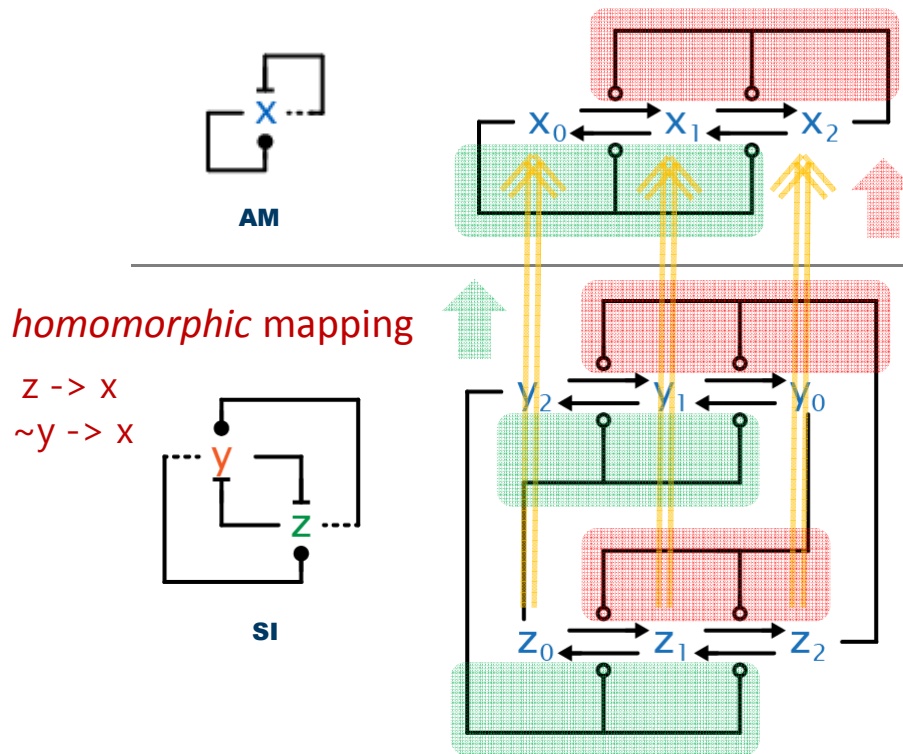
initial conditions:

$$\begin{aligned} z_0 &= y_2 = x_0 \\ z_1 &= y_1 = x_1 \\ z_2 &= y_0 = x_2 \end{aligned}$$

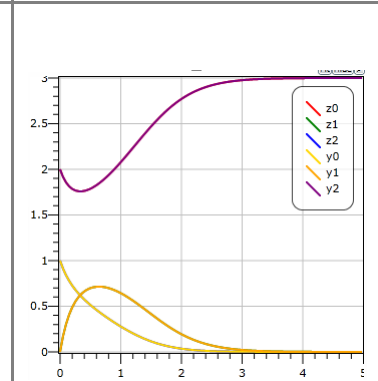
less trivial than you might think:
it need not preserve the out-degree of a node!

Network Emulation: SI emulates AM

A mapping of species and reactions



any initial conditions



initial conditions:

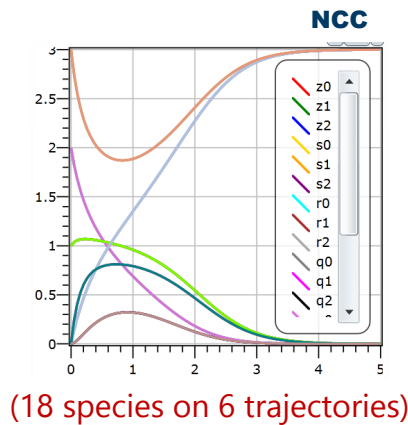
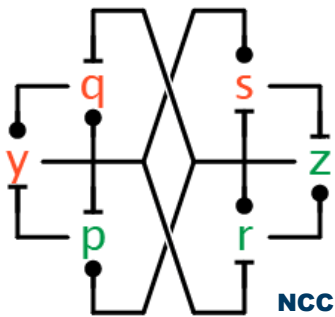
$$Z_0 = Y_2 = X_0$$

$$Z_1 = Y_1 = X_1$$

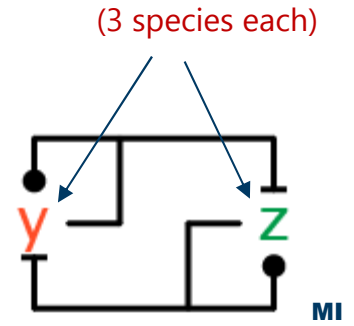
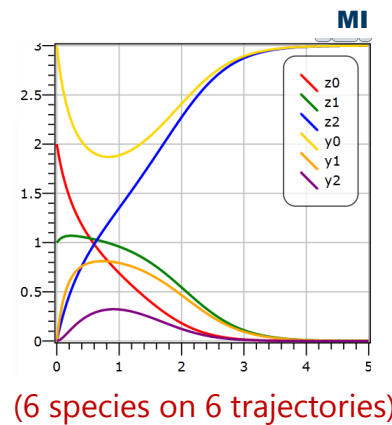
$$Z_2 = Y_0 = X_2$$

Network Emulation: NCC emulates MI

- For *any* rates and initial conditions of MI we can find *some* rates and initial conditions of NCC such that the (18) trajectories of NCC retrace those (6) of MI



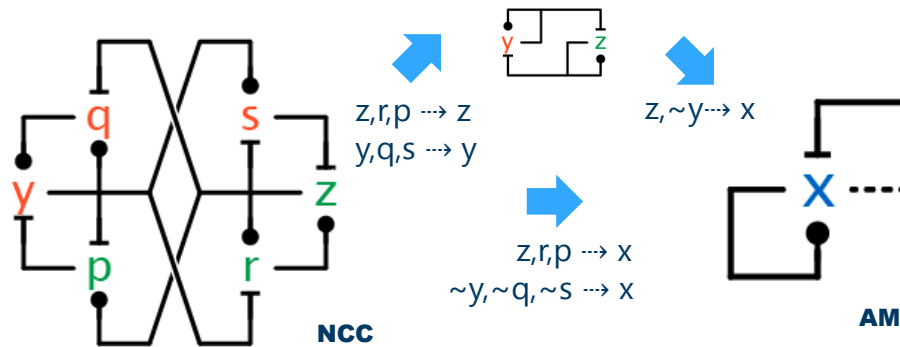
$z, r, p \mapsto z$
 $y, q, s \mapsto y$



initialize
 $z, r, p = z$
 $y, q, s = y$

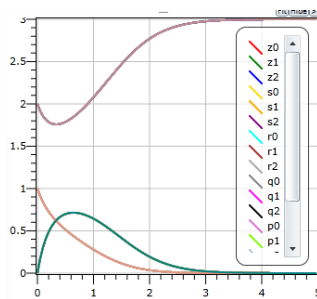
Emulations Compose

- The (18) trajectories NCC can *always* retrace those (3) of AM

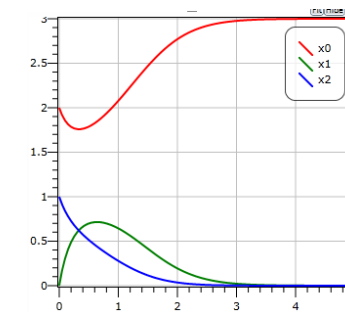


The new cell cycle switch can emulate AM *exactly*.
For *any* initial conditions of AM.

And for *any* rates of AM.

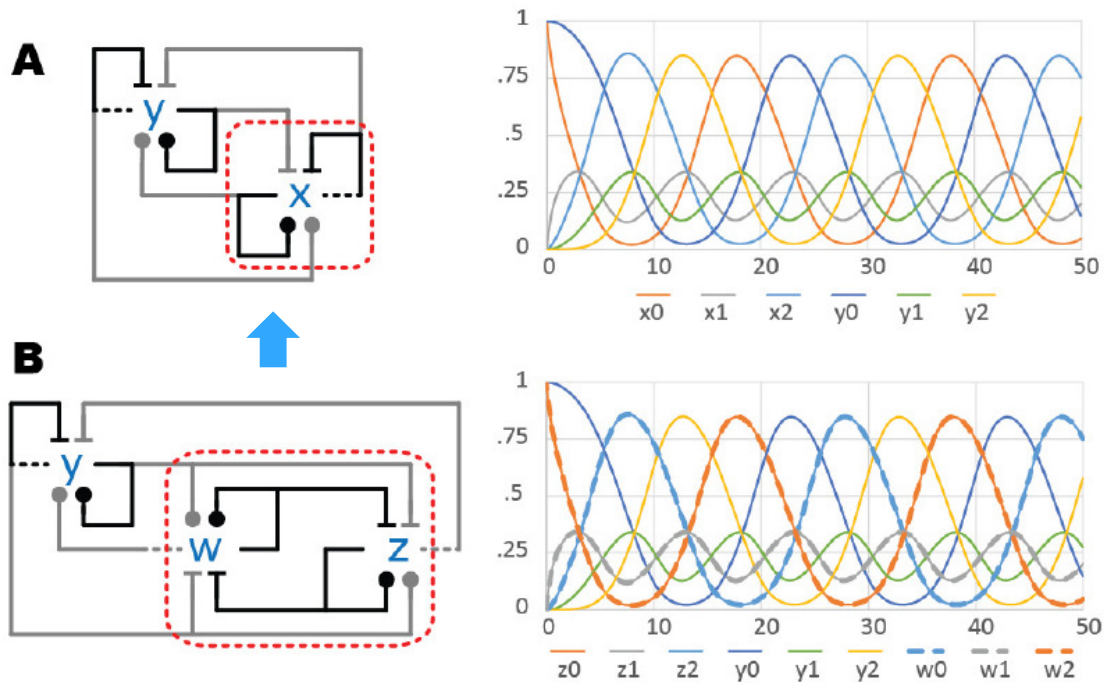


(18 species on 3 trajectories)

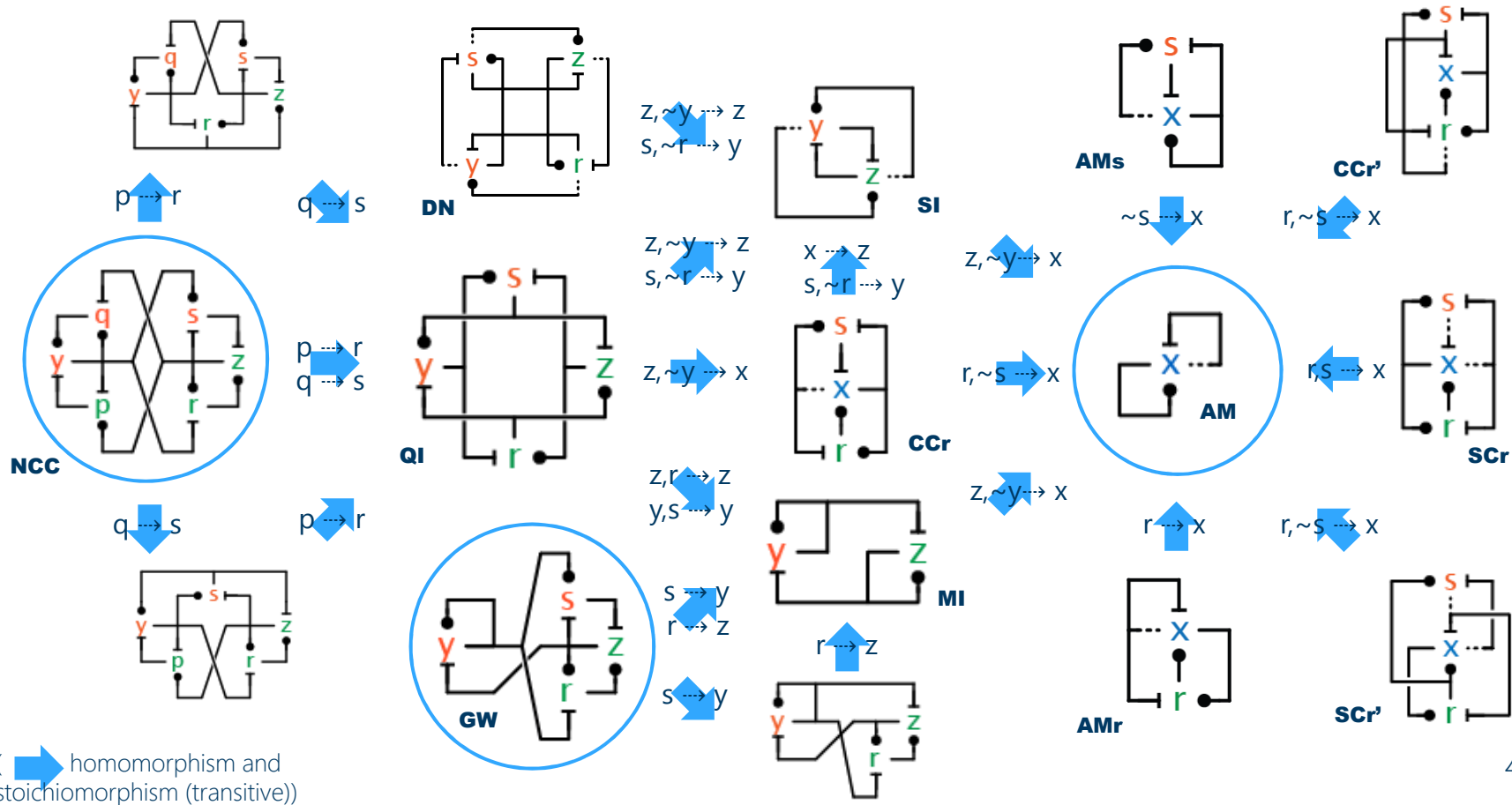


(3 species on 3 trajectories)

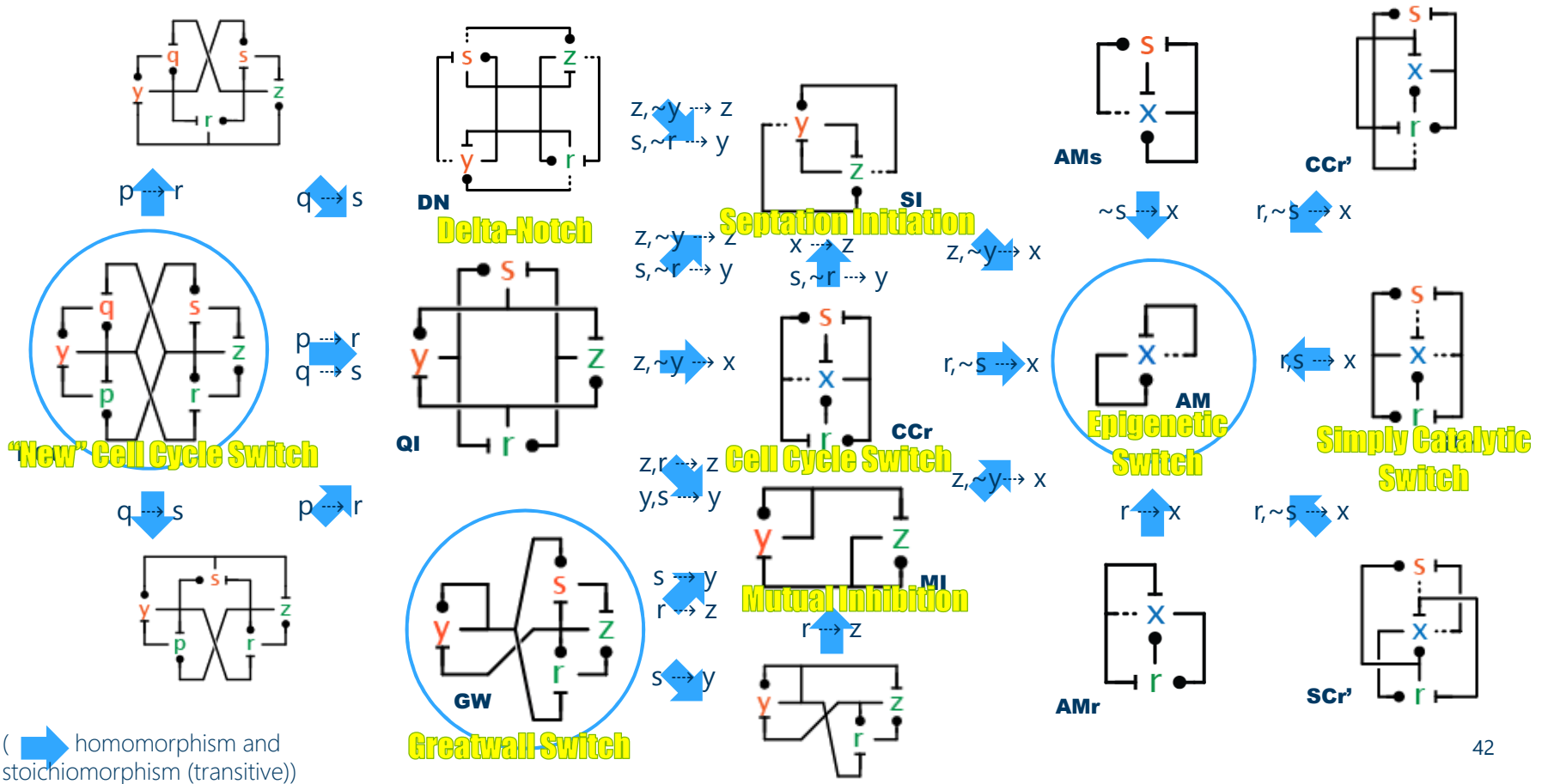
Emulations are Modular



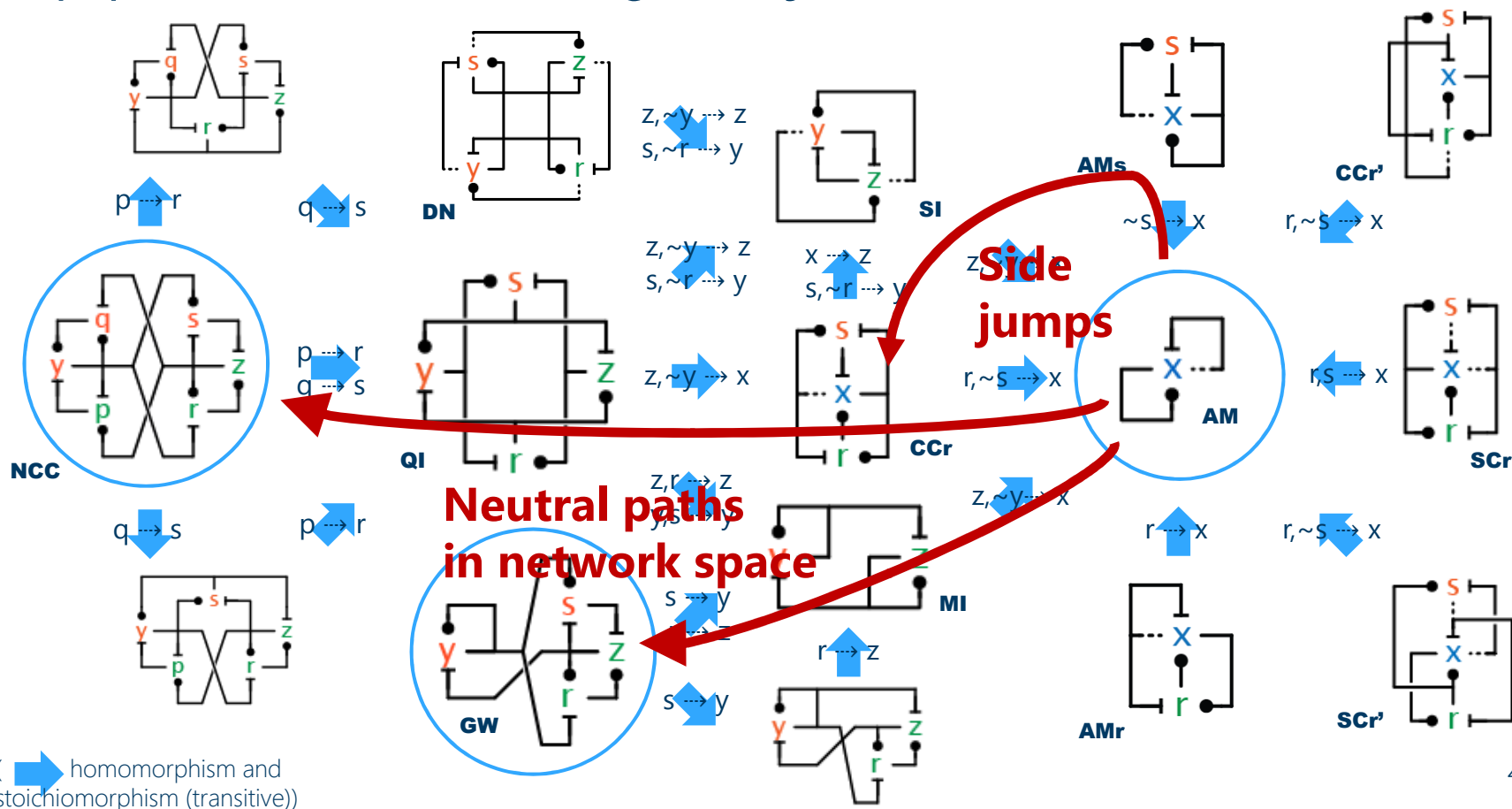
Approximate Majority Emulation Zoo



Approximate Majority Emulation Zoo

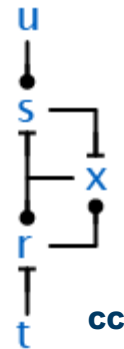
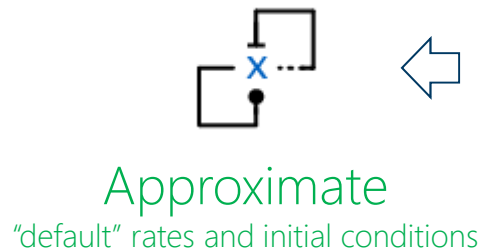


Approximate Majority Emulation Zoo

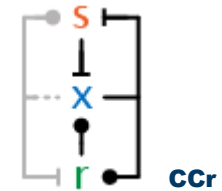
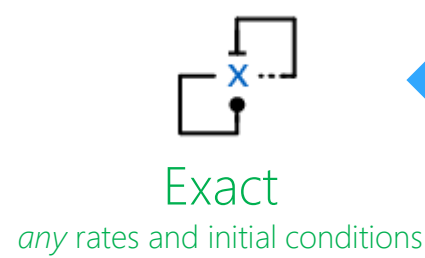


Nature likes a good algorithm

First part

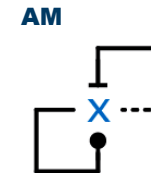
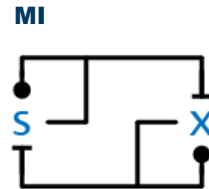
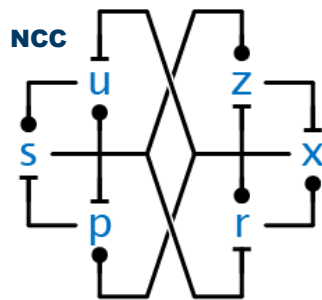


Second part



These additional feedbacks *do exist* in real cell cycles (via indirections)

The cell cycle switch *can exactly* emulate AM



Other work

Characterization of Emulation

Emulation Theorem: If $m \in (S, R) \rightarrow (\hat{S}, \hat{R})$ is a CRN reactant morphism and stoichiomorphism then it is a CRN emulation

reactant morphism $\mathbf{m}_S^T \cdot \boldsymbol{\rho} = \hat{\boldsymbol{\rho}} \cdot \mathbf{m}_R^T$ preserve enough network structure
 stoichiomorphism $\boldsymbol{\varphi} \cdot \mathbf{m}_R = \mathbf{m}_S \cdot \hat{\boldsymbol{\varphi}}$ preserve enough chemical stoichiometry
 \Downarrow
 emulation $\forall \hat{\mathbf{v}}. F(\hat{\mathbf{v}} \circ \mathbf{m}_S) = \hat{F}(\hat{\mathbf{v}}) \circ \mathbf{m}_S$ preserve derivatives

F is the differential system of (S, R) , given by the law of mass action, $\hat{\mathbf{v}}$ is a state of (\hat{S}, \hat{R}) . $\boldsymbol{\varphi}$ is the stoichiometric matrix and $\boldsymbol{\rho}$ is the related reactant matrix. \mathbf{m}_S and \mathbf{m}_R are the characteristic 0-1 matrices of the morphism maps \mathbf{m}_S (on species) and \mathbf{m}_R (on reactions). $-^T$ is transpose. Homomorphism implies reactant morphism.

Cardelli BMC Systems Biology 2014, 8:84
<http://www.biomedcentral.com/1752-0509/8/84>

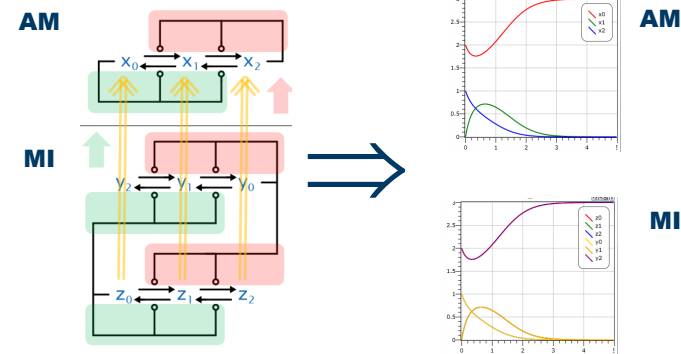


RESEARCH ARTICLE

Open Access

Morphisms of reaction networks that couple structure to function

Luca Cardelli^{1,2}



Model Reduction

- Efficient algorithms to:
 - Discover emulation morphisms
 - Find reduced networks
 - Compute quotient CRNs

Aggregation
reduction

Emulation
reduction

<i>Id</i>	<i>Original model</i>		<i>Forward reduction</i>				<i>Backward reduction</i>			
	<i> R </i>	<i> S </i>	<i>Red.(s)</i>	<i> R </i>	<i> S </i>	<i>Speed-up</i>	<i>Red.(s)</i>	<i> R </i>	<i> S </i>	<i>Speed-up</i>
M1	3538944	262146	4.61E+4	990	222	—	7.65E+4	2708	222	—
M2	786432	65538	1.92E+3	720	167	—	3.68E+3	1950	167	—
M3	172032	16386	8.15E+1	504	122	1.16E+3	1.77E+2	1348	122	5.34E+2
M4	48	18	1.00E-3	24	12	1.00E+0	2.00E-3	45	12	1.00E+0
M5	194054	14531	3.72E+1	142165	10855	1.03E+0	1.32E+3	93033	6634	1.03E+0
M6	187468	10734	3.07E+1	57508	3744	1.92E+1	2.71E+2	144473	5575	3.53E+0
M7	32776	2506	1.26E+0	16481	1281	6.23E+0	1.66E+1	32776	2506	x
M8	41233	2562	1.12E+0	33075	1897	1.12E+0	1.89E+1	41233	2562	x
M9	5033	471	1.91E-1	4068	345	1.04E+0	4.35E-1	5033	471	x
M10	5797	796	1.61E-1	4210	503	1.47E+0	7.37E-1	5797	796	x
M11	5832	730	3.89E-1	1296	217	1.32E+1	6.00E-1	2434	217	7.55E+0
M12	487	85	2.00E-3	264	56	1.88E+0	6.00E-3	426	56	1.31E+0
M13	24	18	1.20E-2	24	18	x	7.00E-3	6	3	1.00E+0

Forward and Backward Bisimulations for Chemical Reaction Networks

Luca Cardelli¹, Mirco Tribastone², Max Tschaikowski³, and Andrea Vandin⁴

¹ Microsoft Research & University of Oxford, UK
luca@microsoft.com

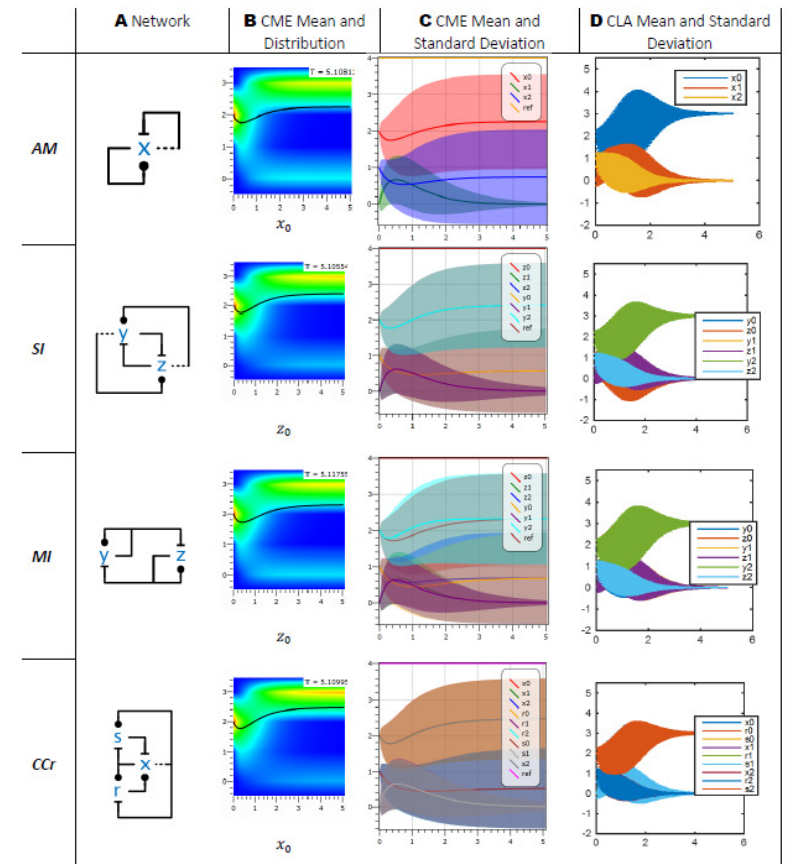
²⁻⁴ University of Southampton, UK
{m.tribastone,m.tschaikowski,a.vandin}@soton.ac.uk

From the BioNetGen database

Stochastic Switches

- Disentangle the contribution of complexity to stochasticity
 - Compare network noise on the baseline of deterministic emulation, across networks of different size and structure

Luca Cardelli, Attila Csikász-Nagy, Neil Dalchau,
Mirco Tribastone, Max Tschaikowski



Synthetic Implementation of AM

- We produced a chemical implementation of AM using DNA gates
- I.e., a 'synthetic reimplementaion' of the central cell-cycle switch.



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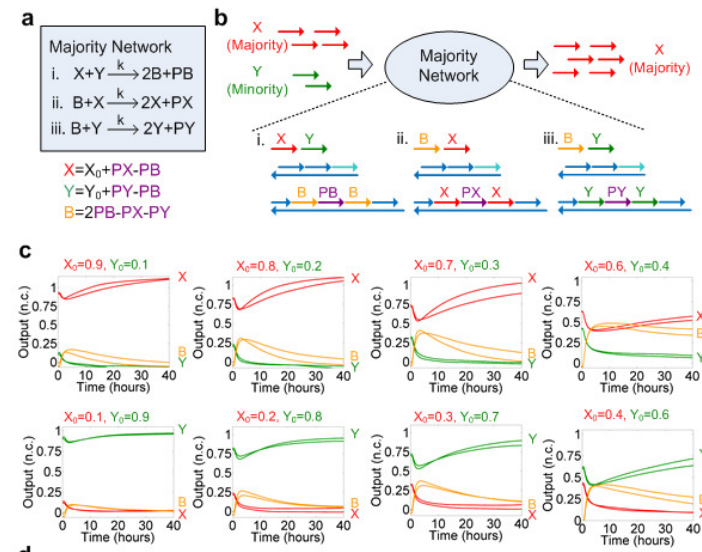
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NATURE NANOTECHNOLOGY | ARTICLE



Programmable chemical controllers made from DNA

Yuan-Jyue Chen, Neil Dalchau, Niranjan Srinivas, Andrew Phillips, Luca Cardelli, David Soloveichik & Georg Seelig



Conclusions

Networks are Algorithms

- They are *methods* for achieving a function
 - We need to understand how these methods relate to each other
 - In addition to how and how well they implement function
 - Algorithms can be obfuscated, and nature can obfuscate networks (to what end?)
- Network emulation can be checked *statically*
 - By stoichiometric/reaction-rate (*structural*) properties
 - That is, no need to compare ODE (*functional*) properties
 - For *any* initial conditions and rates of (one of) the networks
- Efficient algorithms can find emulations
 - Automatic model reduction of large networks